

# Application of Hierarchical Clustering to Find Expression Modules in Cancer

T. M. Murali

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# Innovative Application of Hierarchical Clustering

- ▶ *A module map showing conditional activity of expression modules in cancer*, Eran Segal, Nir Friedman, Daphne Koller and Aviv Regev, Nature Genetics 36, 1090–1098, 2004
- ▶ Analyse gene expression data to find groups of genes expressed in concert between different cancers.
- ▶ Use hierarchical clustering innovatively.

# Goals

- ▶ Move away from standard approach: determine genes that respond (based on cut-off) and study these genes further.
- ▶ Develop method that can analyse large numbers (1000s) of samples across multiple conditions.

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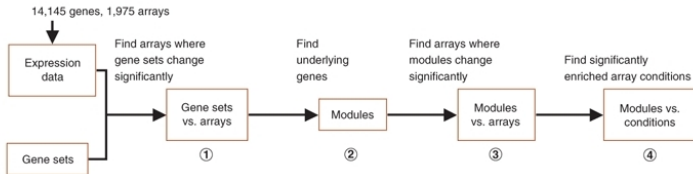
- ▶ Move away from standard approach: determine genes that respond (based on cut-off) and study these genes further.
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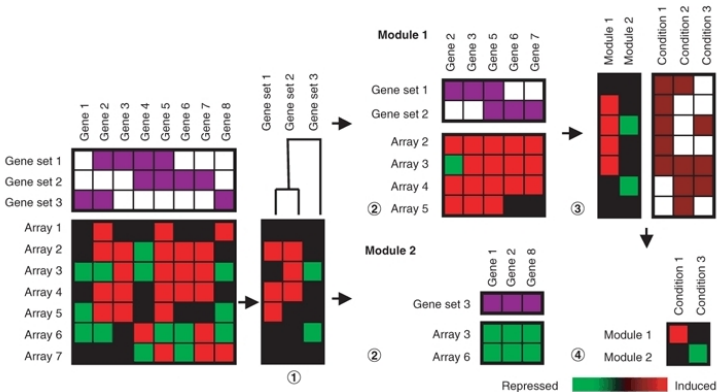
- ▶ Move away from standard approach: determine genes that respond (based on cut-off) and study these genes further.
- ▶ Develop method that can analyse large numbers (1000s) of samples across multiple conditions.
- ▶ Patterns of co-expression across all conditions are not very interesting.
- ▶ Compute *gene modules*: groups of genes that show concerted behaviour across multiple conditions.
- ▶ Specifically, Segal et al. associate with each gene module, a set of samples in which the module is up-regulated and a set of samples in which the module is down-regulated.

# Key Steps

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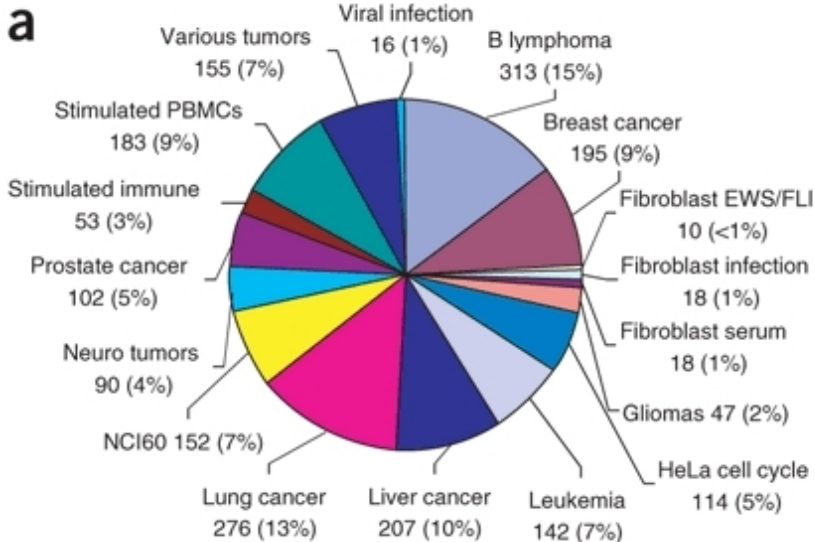
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## Key Steps

- ▶ Group genes into predefined *gene sets*, e.g., groups of genes with the same functional annotation.
- ▶ Convert gene-by-array matrix into gene-set-by-array matrix.
- ▶ Hierarchically cluster gene sets in this matrix.
- ▶ Identify “interesting” gene set clusters (nodes) in the tree.
- ▶ In each gene set cluster, remove genes not expressed consistently with the cluster.

# Gene Expression Data Sets





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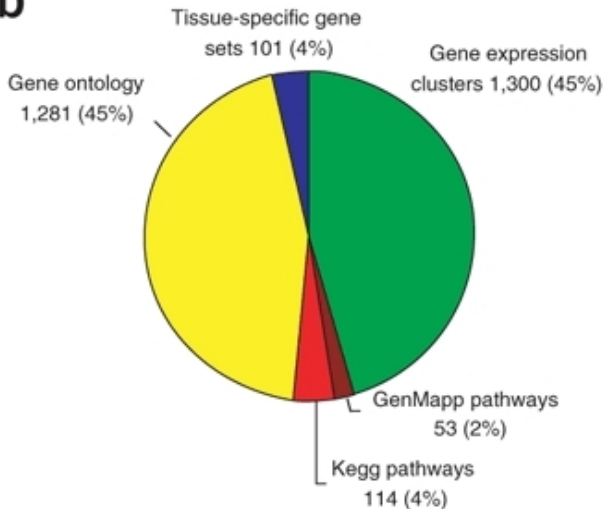
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## Pre-defined Genes Sets

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- ▶ A gene is *induced* (respectively, *repressed*) in an array if its change in expression is  $\geq 2$  (respectively,  $\leq -2$ ).
- ▶ For each gene set-array pair, compute the fraction of genes induced or repressed.
- ▶ Use these values in the gene-set-by-array matrix.

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  - ▶ Statistical test: for a given array, is the fraction of induced genes in a gene set much larger than the fraction of induced genes in the entire array?
  - ▶ Compute the  $p$ -value (statistical significance) of the fraction.
- Exercise.
- ▶ Do so for every gene-set-array pair.
  - ▶ Use false discovery rate correction to account for multiple hypotheses testing.
  - ▶ Replace insignificant entries by 0.

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- ▶ Which nodes do we select as clusters in the tree?
  - ▶ Associate each interior node with Pearson correlation between the two children.
  - ▶ Cluster  $\equiv$  node whose Pearson correlation differs by more than 0.05 from the Pearson correlation of its parent.

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- ▶ Each cluster is a gene set and a set of arrays.
  - ▶ The gene set in a cluster is the union of descendant gene sets (leaves).
  - ▶ The arrays in a cluster are only those that are induced or repressed in the cluster.
- ▶ Module  $\equiv$  Cluster minus genes whose expression is not consistent with the rest of the cluster.



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- ▶ An array contributes to the score only if  $g$  is consistent with  $G$  in the array.
- ▶ Larger contribution from arrays with fewer induced genes.
- ▶ Compute statistical significance of this score.

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- ▶ Under the null hypothesis,  $\text{Score}(g)$  has mean  $\sum_{a \in I \cup R} -p_a \log p_a$  and variance  $\sum_{a \in I \cup R} p_a(1 - p_a) \log^2 p_a$ .

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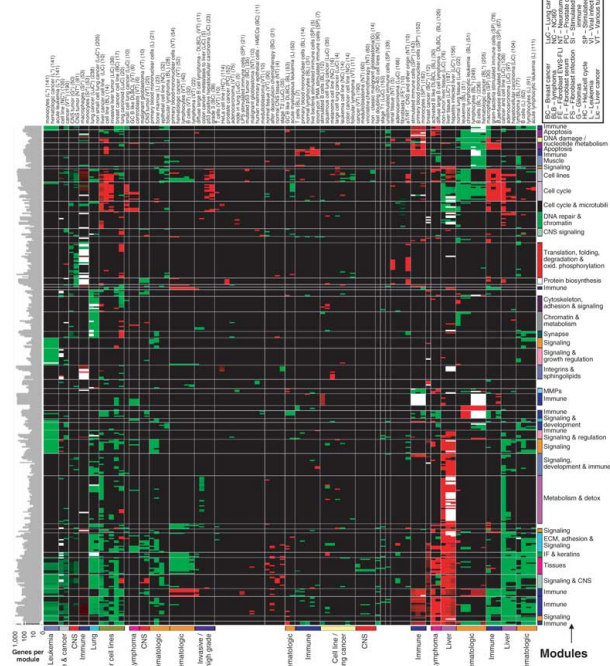
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- ▶ Suppose we observe a score of  $t$ . What is the probability of achieving a score of  $t$  or higher under the null hypothesis? **Exercise.**

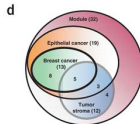
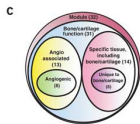
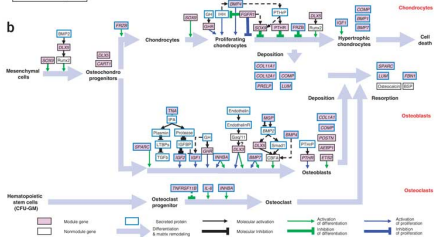
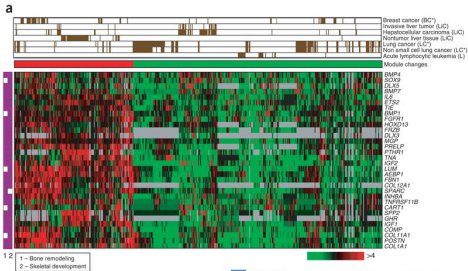
## Further Analysis

- ▶ Statistical significance of computed modules using leave-one-out cross validation (read supplement).
- ▶ Compute enrichment of clinical annotations of the arrays in a module.
- ▶ Visualisation of modules.
- ▶ Literature-based analysis of modules

# Clinical annotations



# Bone Osteoblastic Module



# Conclusions

- ▶ Used pre-defined gene sets to drive hierarchical clustering algorithm.
- ▶ Remove genes from a cluster of gene sets if the gene's expression profile deviates from the cluster.
- ▶ Automatically decide which arrays are part of a module.
- ▶ Natural segue into lectures on biclustering where we will automatically decide which arrays *and* which genes to include in a bicluster.

# Software Exercise

1. Register for and download Genomica.
2. Use Genomica to compute a module map for the sample data set.
3. Download human Entrez Gene gene sets and gene expression data.
4. Run Genomica on these data sets.
5. Change parameters and run Genomica again. Are the results different?

## Computational Exercises

1. In case of  $d_{min}$ , show that the hierarchical clustering algorithm returns the minimum spanning tree.
2. How can we measure the “useful” biological knowledge that a cluster contains?
3. Given an array, the set of genes induced in that array, and a specific gene set, devise a statistical test to determine if the number of induced genes in the gene set is (much) larger than the number of induced genes in the entire array.
4. Under the null hypothesis,  $\text{Score}(g)$  has mean  $\sum_{a \in I_{UR}} -p_a \log p_a$  and variance  $\sum_{a \in I_{UR}} p_a(1 - p_a) \log^2 p_a$ . Suppose we observe a score of  $t$ . What is the probability of achieving a score of  $t$  or higher under the null hypothesis?
5. How will you modify Genomica to accept a new dataset without performing all computations from scratch?