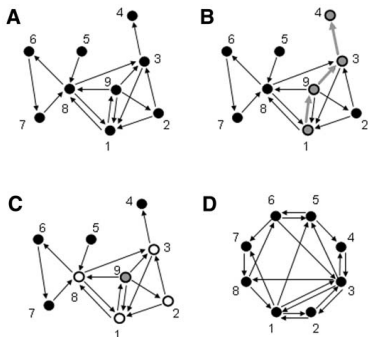


# CS 4884: Small World of Brain Networks



T. M. Murali

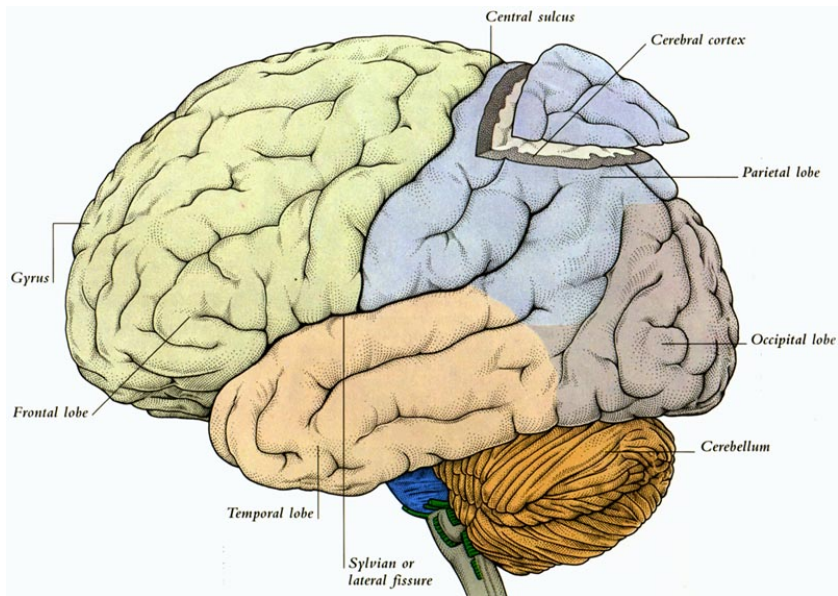
February 13, 2020

# Motivation

- The Watts-Strogatz paper set off a storm of research.
- It has 41,500 citations. Even in 2004, it had more than 2,100 citations.
- The *C. elegans* neuronal network is small-world.

Do mammalian brain networks have the small world property?

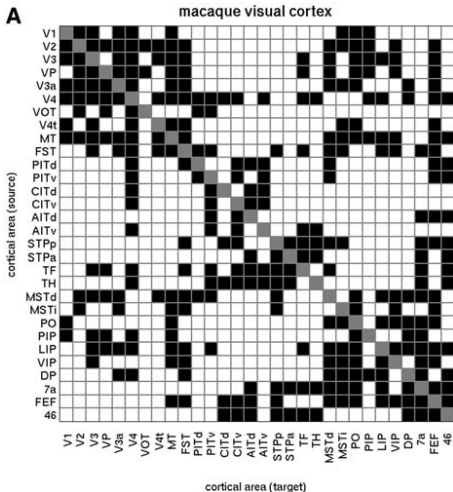
# Visual and Cerebral Cortices



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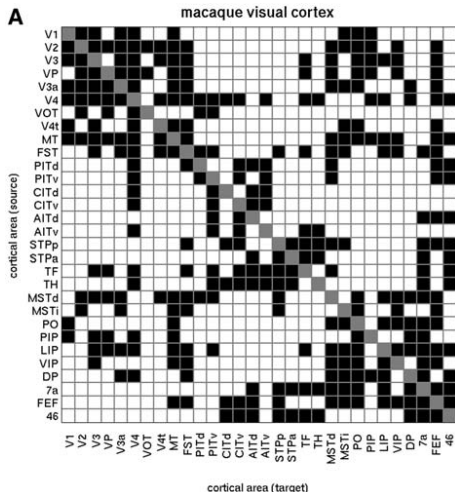


# Datasets: Macaque Visual Cortex



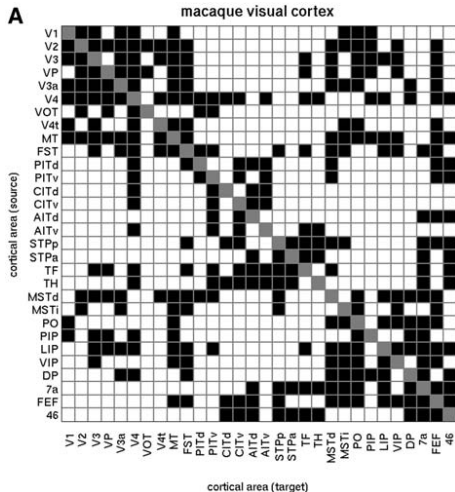
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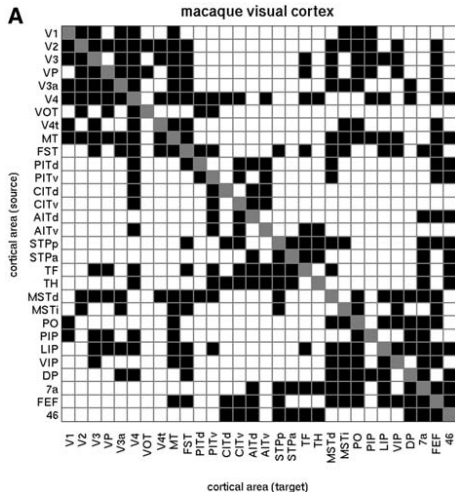
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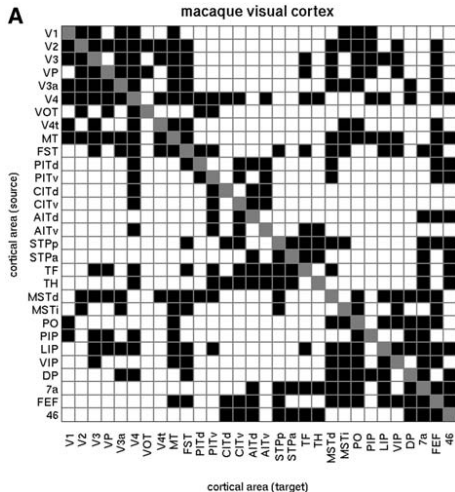
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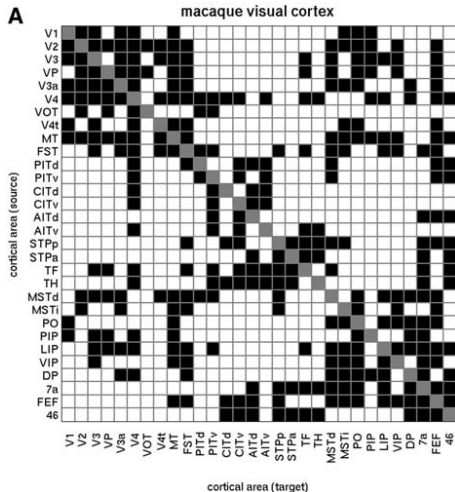


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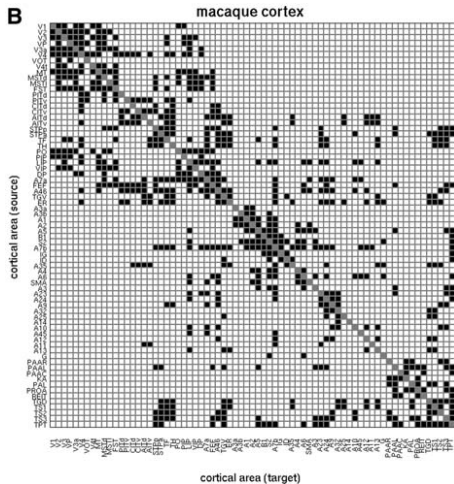
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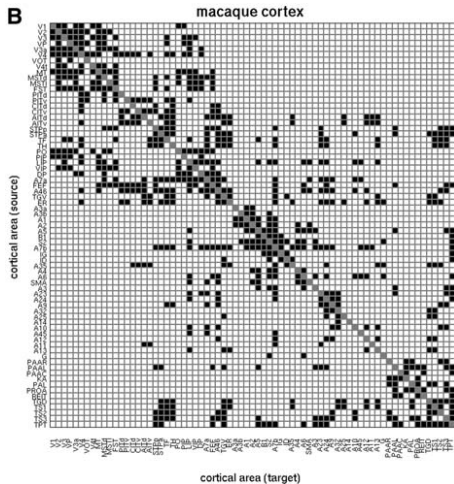
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- $n = 30$ ,  $m = 311$ . (The authors use  $N$  for the number of nodes and  $K$  for the number of edges.)

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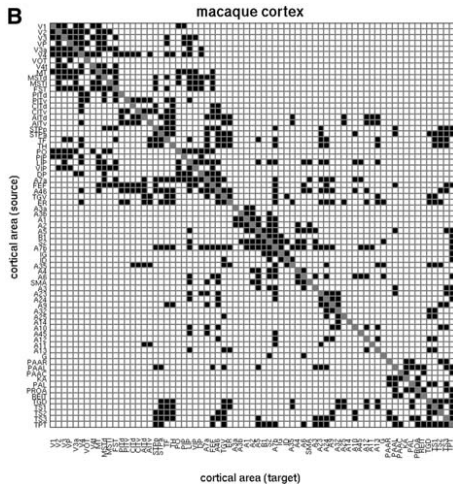
- $n = 71, m = 746$ .

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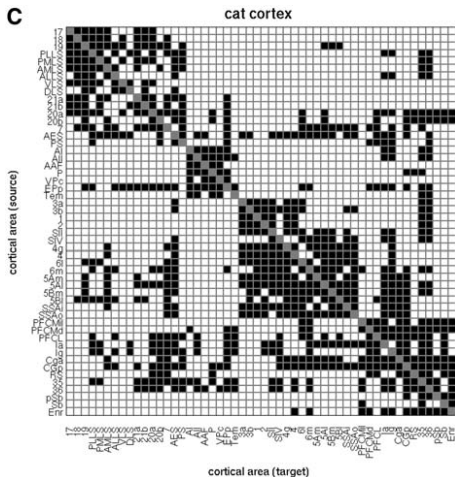
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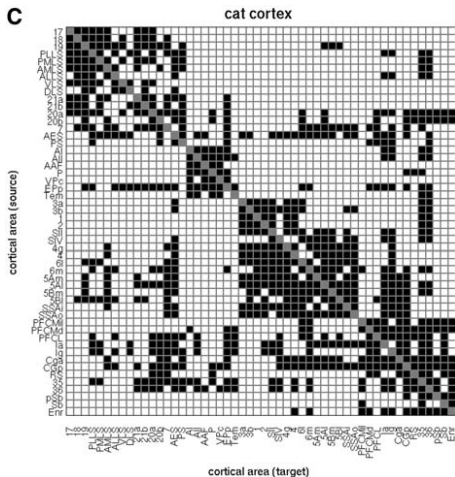
- $n = 71, m = 746$ .
- What is the relation between this graph and the one for the macaque visual cortex? Most of the edges in the previous graph are in this one.

# Datasets: Cat Cerebral Cortex



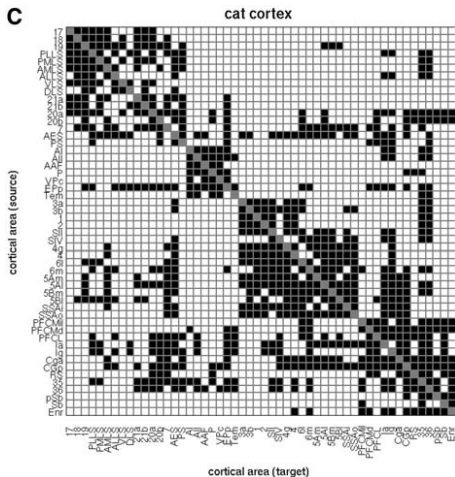
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- $n = 52, m = 820$ .
- What approximation did the authors make? Discarded density information.
- We will ignore density-based connectivity data sets.



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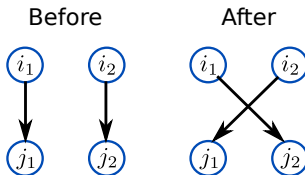
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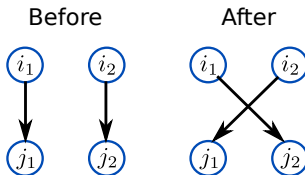
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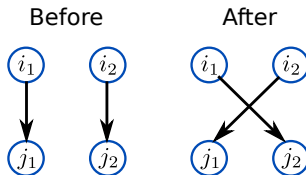
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- Degree-preserving lattice matrix:

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- Degree-preserving lattice matrix: Very poorly specified. Will need to read the code to understand.



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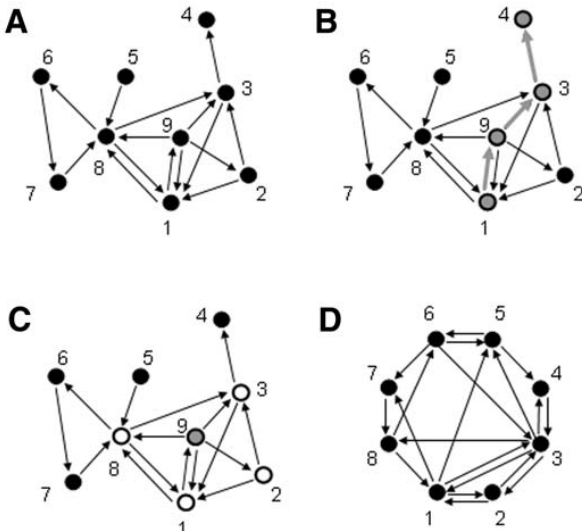
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  - ▶ Clustering coefficient,  $c(v)$ , called “cluster index” and denoted by  $\gamma(v)$  in this paper.

# Computing $l(G)$ and $c(G)$ for Directed Graphs



- Appropriately generalise definitions for undirected graphs.

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- What values will a small world network have for these quantities?  
Small  $l_{\text{scl}}(G)$  and large  $c_{\text{scl}}(G)$ .



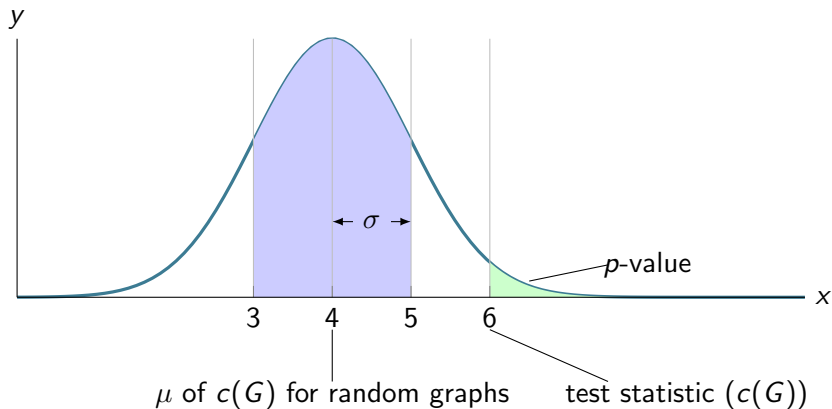
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Table 1. Path Length ( $\lambda$ ,  $\lambda_{scl}$ ) and Cluster Index ( $\gamma$ ,  $\gamma_{scl}$ ) for Large-Scale Connection Matrices of Cortico-Cortical Pathways

<i>Topology</i>	$\lambda$	$\gamma$	$\lambda_{scl}$	$\gamma_{scl}$
MVC	1.7256	0.5313	0.2188	0.5645
$R_{30,311}$	1.6680 (0.0038)*	0.3616 (0.0048) *		
$L_{30,311}$	1.9313 (0.0018)*	0.6622 (0.0000) *		
$Rio_{30,311}$	1.6880 (0.0033)*	0.4305 (0.0059) *		
$Lio_{30,311}$	1.8190 (0.0391)	0.6214 (0.0243)		
MC	2.3769	0.4614	0.1927	0.6117
$R_{71,746}$	2.0310 (0.0051)*	0.1497 (0.0030) *		
$L_{71,746}$	3.8262 (0.0099)*	0.6593 (0.0002) *		
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$Lio_{71,746}$	2.8901 (0.1173)*	0.8992 (0.0211) *		
CC	1.8114	0.5514	0.2498	0.6292
$R_{52,820}$	1.7014 (0.0013)*	0.3103 (0.0026) *		
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$Lio_{71,746}$	2.8901 (0.1173)*	0.8992 (0.0211) *		
CC	1.8114	0.5514	0.2498	0.6292
$R_{52,820}$	1.7014 (0.0013)*	0.3103 (0.0026) *		
$L_{52,820}$	2.1418 (0.0024)*	0.6933 (0.0000) *	< 0.5	> 0.5
$Rio_{52,820}$	1.7217 (0.0037)*	0.4023 (0.0030) *		
$Lio_{52,820}$	1.8570 (0.0283)	0.5893 (0.0172)		

Measures for reference cases represent means and standard deviations (in brackets) for 10 exemplars. Topologies: MVC = macaque visual cortex (Fig. 1A), MC = macaque cortex (Fig. 1B), CC = cat cortex (Fig. 1C),  $R_{N,K}$  = random,  $L_{N,K}$  = lattice,  $Rio_{N,K}$ ,  $Lio_{N,K}$  = random, lattice matrices with in-degree and out-degree distribution preserved. Statistical significance for all comparisons between cortical matrices and random, lattice, Rio, or Lio matrices marked by "\*" are  $p < 0.001$ , the remaining comparisons are  $p < 0.05$ .

## Observations on $l(G)$ and $c(G)$

- Values of  $l(G)$  and  $c(G)$  for connectomes are higher than for random networks with the same number of nodes and edges; difference is statistically significant.
- Conversely, these values for connectomes are lower than for ring networks with the same number of nodes and edges; difference is statistically significant.



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- **Caveats:**
  - ▶  $p$ -values are likely to be underestimated. They should be estimated from empirical distributions built from many more random samples.
  - ▶ No indication of correction for testing multiple hypotheses.
  - ▶ No  $p$ -value associated with the scaled values of  $l(G)$  and  $c(G)$ .

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 $c(G) > c_{\text{random}}$  and  $l(G) > l_{\text{random}}$ , so  $\sigma(G) \approx 1$  or  $\sigma(G) > 1$ .

# Is Small-Worldness Universal in Connectomes?



**OED** Oxford English Dictionary  
The definitive record of the English language

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**connectome, n.** Text size: A A A

View as: [Outline](#) | [Full entry](#) Quotations: [Show all](#) | [Hide all](#) Keywords: [On](#) | [Off](#)

**Pronunciation:** Brit. ▶ /kəˈnɛktəʊm/,  
U.S. ▶ /kəˈnɛk.təʊm/

**Origin:** Formed within English, by compounding. **Etymons:** [CONNECT V.](#), [-OME comb. form.](#)  
**Etymology:** < [CONNECT V.](#) + [-OME comb. form.](#)

The network of nerve cells and their connections found in the brain or other part of the nervous system; a description or map of such a network. Thesaurus >  
Categories >

2005 O. SPORNIS et al. in *PLoS Computational Biol.* (e-journal, accessed 31 July 2018) **1** 0245/1 The purpose of this article is to discuss research strategies aimed at a comprehensive structural description of the network of elements and connections forming the human brain. We propose to call this dataset the human 'connectome'.

2008 *Science* 11 July 181/3 The 'connectome' of the human cortex has been produced by an international team of brain scientists and imagers.

2012 *Guardian* 8 May 10/3 The first detailed connectomes are expected to be completed, and made publicly available for scientists to work on, later this year.

2018 E. C. GOLDFIELD *Bioinspired Devices* vi. 202 The rapid increase in synaptic density, as well as the elongation of axons at around twenty-six weeks, may initiate connectome formation.

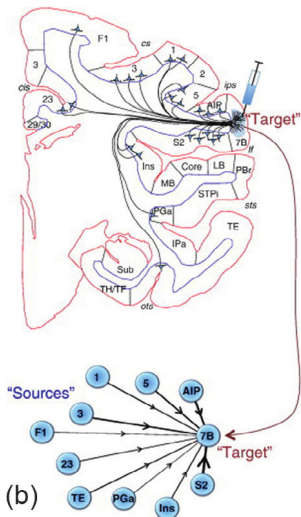
[\(Hide quotations\)](#)

This is a new entry (OED Third Edition, March 2019).  
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- How does small-worldness emerge in connectomes?
- Do all connectomes have the small-world property?

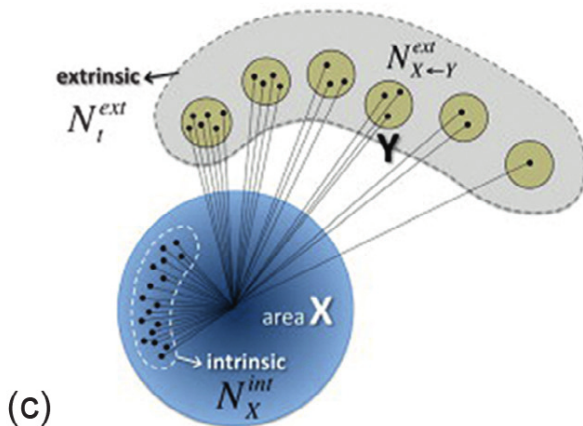


# Structural Connectivity at the Mesoscale: Macaque



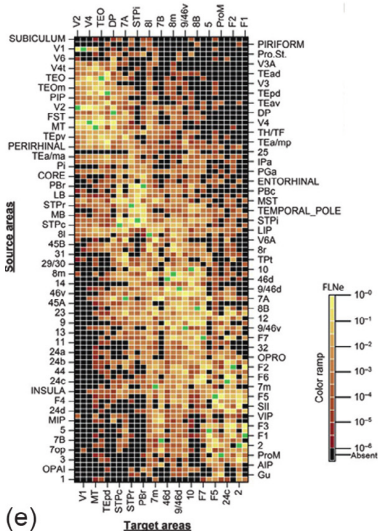
Use retrograde tract tracing. Determine edges coming into node representing area of injection from “labelled” nodes representing neurons that the tracer reaches.

# Structural Connectivity at the Mesoscale: Macaque



Injection is at X:  $w(Y, X) = \frac{\text{number of neurons labelled in } Y}{\text{total number of labelled neurons}}$

# Structural Connectivity at the Mesoscale: Macaque

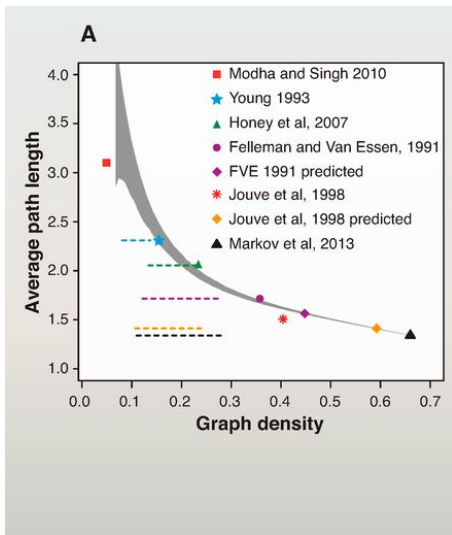


(e)

Example of connectivity matrix.

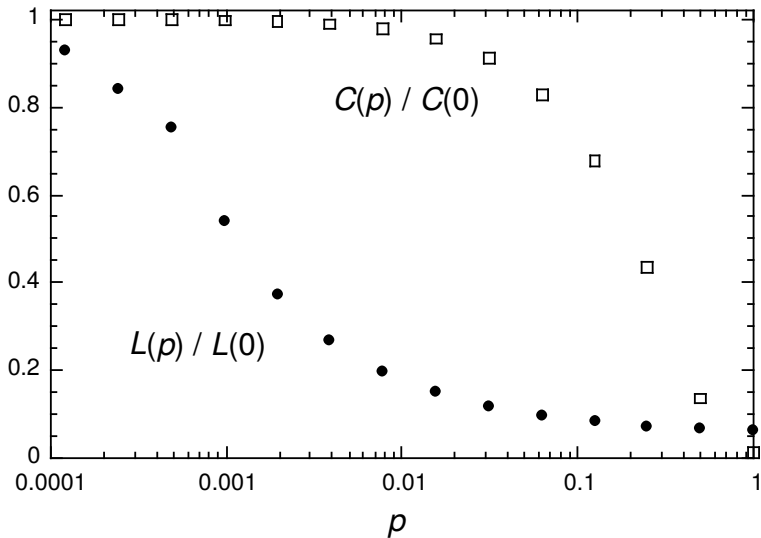
Edge weights range over six orders of magnitude.

# Arguments Against Universality

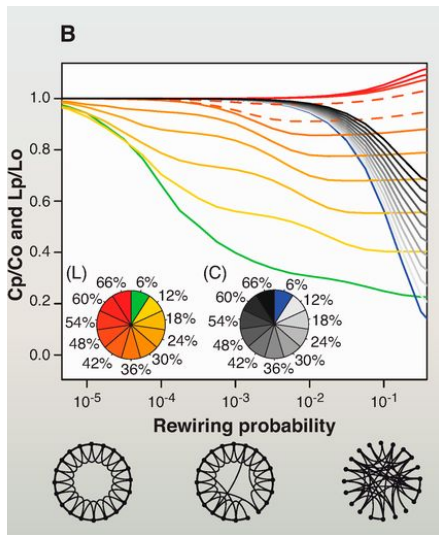


Markov et al., *Cortical high-density counterstream architectures*. *Science*, 2013.

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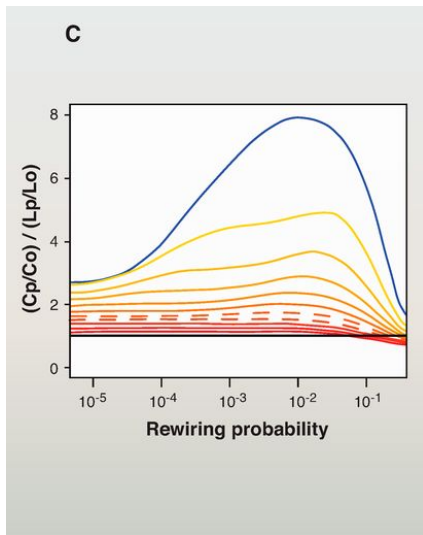


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**Table 1.** Small-World Metrics.<sup>a</sup>

	Macaque		Mouse	
	Binary	Weighted	Binary	Weighted
$\Gamma$	$1.21 \pm 0.014$	$1.59 \pm 0.007$	$1.31 \pm 0.004$	$1.76 \pm 0.009$
$\Lambda$	$1.00 \pm 0.000$	$1.27 \pm 0.057$	$1.00 \pm 0.000$	$1.47 \pm 0.021$
$\sigma$	$1.21 \pm 0.014$	$1.25 \pm 0.071$	$1.31 \pm 0.004$	$1.20 \pm 0.019$
$\phi$	N/A	$0.574 \pm 0.041$	N/A	$0.800 \pm 0.002$

<sup>a</sup>For the macaque and mouse connectomes, we show the mean and standard deviation of the normalized clustering coefficient ( $\Gamma$ ), the normalized path length ( $\Lambda$ ), the small-world index ( $\sigma$ ), and the small-world propensity ( $\phi$ ) for both binary and weighted graphs.